Differentiation

2013

Q2 – 3 marks

Differentiate $f(x) = e^{\cos x}\sin^2 x$. **3**

Marking Instructions

Differentiate $f(x) = e^{\cos x}\sin^2 x$.	**3**	
$f'(x) = e^{\cos x}(-\sin x).\sin^2 x + e^{\cos x}.2\sin x\cos x$		•[1] Uses product rule.[1] •[2] First term correct. •[3] Second term correct.[2] Simplified alternatives.
$= -e^{\cos x}\sin^3 x + e^{\cos x}.2\sin x\cos x$ $= e^{\cos x}(\sin 2x - \sin^3 x)$ $= e^{\cos x}\sin x(2\cos x - \sin^2 x)$		

Notes:

2.1 Evidence of method: Statement of the rule and evidence of progress in applying it.
OR Application showing the ***sum*** of two terms, both involving differentiation.

2.2 Signs switched: •[1] •[3] available for $e^{\cos x}\sin^3 x - e^{\cos x}.2\sin x\cos x$ or equivalent.

2012
Q1 – 7 marks

(*a*) Given $f(x) = \dfrac{3x+1}{x^2+1}$, obtain $f'(x)$. **3**

(*b*) Let $g(x) = \cos^2 x \exp(\tan x)$. Obtain an expression for $g'(x)$ and simplify your answer. **4**

Marking Instructions

(a) $f(x) = \dfrac{3x+1}{x^2+1}$		
$f'(x) = \dfrac{3(x^2+1) - (3x+1)2x}{(x^2+1)^2}$	1M	for quotient rule (or product)
	1	for two correct terms
	1	for third correct term
$= \dfrac{3x^2+3-6x^2-2x}{(x^2+1)^2}$		
$= \dfrac{-3x^2-2x+3}{(x^2+1)^2}$		
(b) $g(x) = \cos^2 x\, e^{\tan x}$		
	1M	product rule
$g'(x) = 2\cos x(-\sin x)e^{\tan x} + (\cos^2 x)(\sec^2 x)e^{\tan x}$	1	first correct term
	1	second correct term
$= -\sin 2x\, e^{\tan x} + e^{\tan x}$	1	simplification
$= (1 - \sin 2x)e^{\tan x}$		
(b) *alternative*		
$g(x) = \cos^2 x \exp(\tan x)$		
$\ln(g(x)) = \ln(\cos^2 x) + \tan x$	1M	
$= 2\ln(\cos x) + \tan x$		
Differentiating		
$\dfrac{g'(x)}{g(x)} = 2\dfrac{(-\sin x)}{\cos x} + \sec^2 x$	1	
$g'(x) = \left(\dfrac{1 - 2\sin x\cos x}{\cos^2 x}\right)\cos^2 x \exp(\tan x)$	1	
$= (1 - \sin 2x)\tan x$	1	

2011
Q3b – 3 marks

(*b*) Given $f(x) = \sin x \cos^3 x$, obtain $f'(x)$. **3**

Marking Instructions

(b) *Method 1*

$f(x) = \sin x \cos^3 x$		
	M1	for using the product rule
$f'(x) = \cos^4 x + \sin x(-3\cos^2 x \sin x)$	**1**	for first term
	1	for second term
$= \cos^4 x - 3\cos^2 x \sin^2 x$		

Method 2

$f(x) = \sin x \cos^3 x$		
$\ln(f(x)) = \ln \sin x + \ln(\cos^3 x)$	**M1**	
$\frac{f'(x)}{f(x)} = \frac{\cos x}{\sin x} - \frac{3\cos^2 x \sin x}{\cos^3 x}$	**1**	
$= \frac{\cos x}{\sin x} - \frac{3\sin x}{\cos x}$		
$f'(x) = \left(\frac{\cos x}{\sin x} - \frac{3\sin x}{\cos x}\right)\sin x \cos^3 x$	**1**	
$= \cos^4 x - 3\sin^2 x \cos^2 x$		

2011
Q7 – 4 marks

A curve is defined by the equation $y = \dfrac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$ for $x < 1$.

Calculate the gradient of the curve when $x = 0$. **4**

Marking Instructions

Working	Mark	Comment
Method 1		
$y = \dfrac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$		
$\Rightarrow \ln y = \ln\left(e^{\sin x}(2+x)^3\right) - \ln\left(\sqrt{1-x}\right)$	**1M**	for use of logs
$= \sin x + 3\ln(2+x) - \frac{1}{2}\ln(1-x)$	**1**	for preparing to differentiate
$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \cos x + \dfrac{3}{2+x} + \dfrac{1}{2(1-x)}$	**1**	
$\dfrac{dy}{dx} = y\left(\cos x + \dfrac{3}{2+x} + \dfrac{1}{2(1-x)}\right)$		
When $x = 0, y = 8 \Rightarrow$		
gradient $= 8\left(1 + \dfrac{3}{2} + \dfrac{1}{2}\right) = 24.$	**1**	for final value

Working	Mark	Comment
Method 2		
$y = \dfrac{e^{\sin x}(2+x)^3}{\sqrt{1-x}} \Rightarrow \dfrac{dy}{dx} =$		
$\dfrac{\frac{d}{dx}\left(e^{\sin x}(2+x)^3\right)\sqrt{1-x} - e^{\sin x}(2+x)^3\left(-\frac{1}{2}\frac{1}{\sqrt{1-x}}\right)}{(1-x)}$	**M1**	for use of quotient rule
$= \dfrac{\left[\cos x\, e^{\sin x}(2+x)^3 + 3e^{\sin x}(2+x)^2\right](1-x)}{(1-x)^{3/2}}$	**M1**	
$+ \dfrac{e^{\sin x}(2+x)^3}{2(1-x)^{3/2}}$	**1**	
When $x = 0$,		
gradient $= \dfrac{(2^3 + 3\times 2^2)}{1} + \dfrac{2^3}{2} = 20 + 4 = 24$	**1**	
Method 3		
$y = \frac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$		
$y\sqrt{1-x} = e^{\sin x}(2+x)^3$	**1**	
$\sqrt{1-x}\frac{dy}{dx} - \frac{1}{2}y(1-x)^{-1/2}$		
$= \cos x\, e^{\sin x}(2+x)^3 + 3e^{\sin x}(2+x)^2$	**1,1**	
when $x = 0, y = \frac{e^0 2^3}{1} = 8$. This leads to		
$\frac{dy}{dx} = 24$	**1**	

2010

Q1 – 6 marks

Differentiate the following functions.

(a) $f(x) = e^x \sin x^2$. **3**

(b) $g(x) = \dfrac{x^3}{(1 + \tan x)}$. **3**

Marking Instructions

(a) For $f(x) = e^x \sin x^2$,		
$f'(x) = e^x \sin x^2 + e^x(2x \cos x^2)$.	**1M**	using the Product Rule
	1,1	one for each correct term
(b) *Method* 1		
For $g(x) = \dfrac{x^3}{(1 + \tan x)}$,		
$g'(x) = \dfrac{3x^2(1 + \tan x) - x^3 \sec^2 x}{(1 + \tan x)^2}$.	**1M**	using the Quotient Rule
	1	first term and denominator
	1	second term
Method 2		
$g(x) = x^3(1 + \tan x)^{-1}$	**1**	for correct rewrite
$g'(x) =$		
$3x^2(1 + \tan x)^{-1} + x^3(-1)(1 + \tan x)^{-2} \sec^2 x$	**1,1**	for accuracy
$= \frac{x^2}{(1 + \tan x)^2}(3 + 3 \tan x - x \sec^2 x)$		

2009

Q1a – 3 marks

(a) Given $f(x) = (x + 1)(x - 2)^3$, obtain the values of x for which $f'(x) = 0$. **3**

Marking Instructions

$f(x) = (x + 1)(x - 2)^3$	
$f'(x) = (x - 2)^3 + 3(x + 1)(x - 2)^2$	**1**
$= (x - 2)^2((x - 2) + 3(x + 1))$	
$= (x - 2)^2(4x + 1)$	**1**
$= 0$ when $x = 2$ and when $x = -\frac{1}{4}$.	**1**

2008
Q2a – 2 marks

(*a*) Differentiate $f(x) = \cos^{-1}(3x)$ where $-\frac{1}{3} < x < \frac{1}{3}$. **2**

Marking Instructions

$f(x) = \cos^{-1}(3x)$

$f'(x) = \dfrac{-1}{\sqrt{1-(3x)^2}} \times 3$ **1,1**

$= \dfrac{-3}{\sqrt{1-9x^2}}$

2008
Q15 – 9 marks

Let $f(x) = \dfrac{x}{\ln x}$ for $x > 1$.

(*a*) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers. **2,2**

(*b*) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$. **3**

(*c*) Obtain the coordinates of the point of inflexion. **2**

Marking Instructions

(a) $\dfrac{d}{dx}\left(\dfrac{x}{\ln x}\right) = \dfrac{1 \times \ln x - x \times \frac{1}{x}}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$. **1M,1**

$\dfrac{d^2}{dx^2}\left(\dfrac{x}{\ln x}\right) = \dfrac{\frac{1}{x} \times (\ln x)^2 - (\ln x - 1) \times \frac{2\ln x}{x}}{(\ln x)^4}$ **1**

$= \dfrac{\ln x - 2\ln x + 2}{x(\ln x)^3} = \dfrac{2 - \ln x}{x(\ln x)^3}$ **1**

(b) Stationary points when $\ln x = 1$, giving $x = e$ and $y = e$. **1**

At (e, e), the second derivative is

$\dfrac{2-1}{e \times 1^3} > 0$ **1**

so (e, e) is a minimum. **1**

(c) When $\frac{d^2y}{dx^2} = 0$, $\ln x = 2 \Rightarrow x = e^2$. **1**

$x = e^2 \Rightarrow y = \frac{1}{2}e^2$. **1**

2007
Q2a – 3 marks

Obtain the derivative of each of the following functions:

(*a*) $f(x) = \exp(\sin 2x)$; **3**

Marking Instructions

$f(x) = \exp(\sin 2x)$

$f'(x) = 2\cos 2x \exp(\sin 2x)$ **M1,2E1**