Euclidean Algorithm

2013

Q5 – 4 marks

Use the Euclidean algorithm to obtain the greatest common divisor of 1204 and 833, expressing it in the form $1204a + 833b$, where a and b are integers. **4**

Marking Instructions

$1204 = 1 \times 833 + 371$	•[1] Starting correctly.
$833 = 2 \times 371 + 91$ $371 = 4 \times 91 + 7$ $91 = 13 \times 7$ so gcd is 7	•[2] Obtains GCD. Accept $(833, 1204) = 7$
$7 = 371 - 4 \times 91$ $= 371 - 4(833 - 2 \times 371)$ $= 9 \times 371 - 4 \times 833$ $= 9(1204 - 1 \times 833) - 4 \times 833$ $= 9 \times 1204 - 13 \times 833$	•[3] Equates GCD from •[2] and evidence of correct back substitution.[1,4]
$(a = 9, b = -13)$	•[4] Correct form of final answer.[5]

2009

Q10 – 4 marks

Use the Euclidean algorithm to obtain the greatest common divisor of 1326 and 14654, expressing it in the form $1326a + 14654b$, where a and b are integers. **4**

Marking Instructions

$14654 = 11 \times 1326 + 68$	1
$1326 = 19 \times 68 + 34$	
$68 = 2 \times 34$	1
$34 = 1326 - 19 \times 68$	1
$= 1326 - 19(14654 - 11 \times 1326)$	
$= 210 \times 1326 - 19 \times 14654$	1

2007

Q7 – 4 marks

Use the Euclidean algorithm to find integers p and q such that $599p + 53q = 1$. **4**

Marking Instructions

$$599 = 53 \times 11 + 16$$
$$53 = 16 \times 3 + 5$$
$$16 = 5 \times 3 + 1 \quad \mathbf{1}$$

$$\begin{aligned} 1 &= 16 - 5 \times 3 \\ &= 16 - (53 - 16 \times 3) \times 3 \\ &= 16 \times 10 - 53 \times 3 \\ &= (599 - 53 \times 11) \times 10 - 53 \times 3 \\ &= 599 \times 10 - 53 \times 113 \end{aligned} \quad \mathbf{2E1}$$

Hence $599p + 53q = 1$ when $p = 10$ and $q = -113$. **1**