

Partial Fractions

2012

Q15(a) – 4 marks

Express $\frac{1}{(x-1)(x+2)^2}$ in partial fractions.

Marking Instructions

$$\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \quad \mathbf{1M}$$

$$1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$$x = 1 \Rightarrow A = \frac{1}{9} \quad \mathbf{1}$$

$$x = -2 \Rightarrow C = -\frac{1}{3} \quad \mathbf{1}$$

$$x = 0 \Rightarrow 1 = \frac{4}{9} - 2B + \frac{1}{3} \Rightarrow B = -\frac{1}{9} \quad \mathbf{1}$$

$$\therefore \frac{1}{(x-1)(x+2)^2} = \frac{1}{9} \left(\frac{1}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2} \right)$$

2011

Q1 (a bit of) – 3 marks

Express $\frac{13-x}{x^2+4x-5}$ in partial fractions

Marking Instructions

$$\frac{13-x}{x^2+4x-5} = \frac{13-x}{(x-1)(x+5)}$$

$$= \frac{A}{x-1} + \frac{B}{x+5} \quad \mathbf{1}$$

$$13-x = A(x+5) + B(x-1)$$

$$x = 1 \Rightarrow 12 = 6A \Rightarrow A = 2 \quad \mathbf{1} \quad \text{for first value}$$

$$x = -5 \Rightarrow 18 = -6B \Rightarrow B = -3 \quad \mathbf{1} \quad \text{for second value}$$

$$\text{Hence } \frac{13-x}{x^2+4x-5} = \frac{2}{x-1} - \frac{3}{x+5}$$

2010

Q7 (a bit of) – 4 marks

Express $\frac{3x+5}{(x+1)(x+2)(x+3)}$ in partial fractions.

Marking Instructions

$\frac{3x+5}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$	1M	
$3x + 5 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$		
$x = -1 \Rightarrow 2 = 2A \Rightarrow A = 1$	1	for first correct coefficient
$x = -2 \Rightarrow -1 = -B \Rightarrow B = 1$	1	for second correct coefficient
$x = -3 \Rightarrow -4 = 2C \Rightarrow C = -2$		
Hence		for last coefficient and applying them
$\frac{3x+5}{(x+1)(x+2)(x+3)} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{2}{x+3}$	1	

2009

Q14 (a bit of) – 4 marks

Express $\frac{x^2+6x-4}{(x+2)^2(x-4)}$ in partial fractions.

Marking Instructions

$\frac{x^2+6x-4}{(x+2)^2(x-4)} = \frac{A}{(x+2)^2} + \frac{B}{x+2} + \frac{C}{x-4}$	M1	
$x^2+6x-4 = A(x-4) + B(x+2)(x-4) + C(x+2)^2$		
Let $x = -2$ then $4 - 12 - 4 = -6A \Rightarrow A = 2$.	1	
Let $x = 4$ then $16 + 24 - 4 = 36C \Rightarrow C = 1$.	1	
Let $x = 0$ then		
$-4 = -4A - 8B + 4C \Rightarrow -4 = -8 - 8B + 4 \Rightarrow B = 0$.	1	
Thus		
$\frac{x^2+6x-4}{(x+2)^2(x-4)} = \frac{2}{(x+2)^2} + \frac{1}{x-4}$		

2005

Q13 (a bit of) – 4 marks

Express $\frac{1}{x^3 + x}$ in partial fractions.

Marking Instructions

$$\begin{aligned}\frac{1}{x^3 + x} &= \frac{A}{x} + \frac{Bx + C}{x^2 + 1} && 1 \\ 1 &= A(x^2 + 1) + (Bx + C)x \\ x = 0 &\Rightarrow 1 = A \Rightarrow A = 1 && 1 \\ x = 1 &\Rightarrow 1 = 2 + B + C \\ x = -1 &\Rightarrow 1 = 2 + B - C \\ &\Rightarrow C = 0, B = -1 && 1,1 \\ \frac{1}{x^3 + x} &= \frac{1}{x} - \frac{x}{x^2 + 1}\end{aligned}$$

2004

Q5 (a bit of) – 2 marks

Express $\frac{1}{x^2 - x - 6}$ in partial fractions.

Marking Instructions

$$\begin{aligned}\frac{1}{x^2 - x - 6} &= \frac{A}{x - 3} + \frac{B}{x + 2} && 1 \text{ for method} \\ &= \frac{1}{5(x - 3)} - \frac{1}{5(x + 2)} && 1\end{aligned}$$

2002

A8 (a bit of) – 3 marks

Express $\frac{x^2}{(x+1)^2}$ in the form $A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, ($x \neq -1$), stating the values of the constants A , B and C .

Marking Instructions

$$\begin{aligned}\frac{x^2}{(x+1)^2} &= A + \frac{B}{x+1} + \frac{C}{(x+1)^2} \text{ so} \\ x^2 &= A(x+1)^2 + B(x+1) + C \\ &= Ax^2 + (2A+B)x + A+B+C \\ \text{Hence } A &= 1, B = -2 \text{ and } C = 1. \\ \text{(a) } y &= 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}\end{aligned}$$

1 for valid method

2E1 for the values