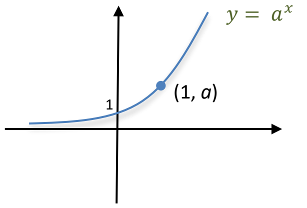
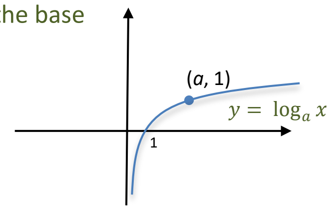
ZETA MATHS

Higher Mathematics
Revision Checklist

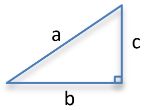
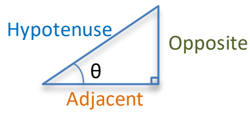
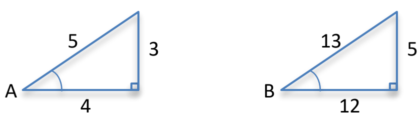
Contents:

V.1.3

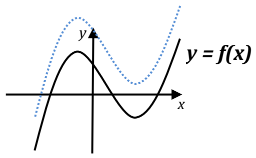
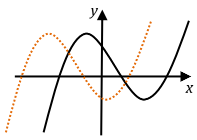
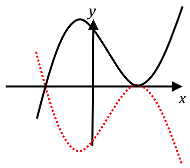
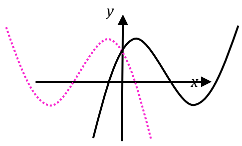
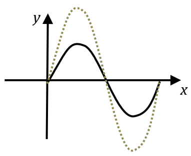
Expressions & Functions

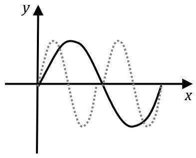
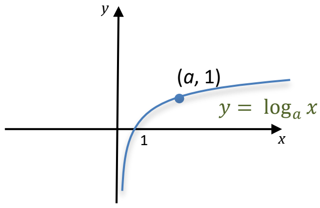
Topic	Skills	Notes			
Logs and Exponentials					
Prior Skills					
Equation of a Line	Know and use *y* = m*x* + c to determine the equation of a line				
New Skills					
Exponential Functions	An exponential function is written in the form $y = a^x$ where *a* is the base, and *x* is the index or exponent  				
The Logarithmic Function	The logarithmic function is the inverse of the exponential function. It is written as $y = \log_a x$ where *a* is the base **NB:** On your calculator the *log* button is $\log_{10} x$  				
Convert Between Logarithmic and Exponential Form	If $y = a^x$ then $x = \log_a y$ **e.g.** $3 = \log_a 8$ $a^3 = 8$ $a = 2$				
The Exponential Function	The exponential function is written as $y = e^x$ where *e* is the base, which is approximately 2.718				
Natural Logarithms	The natural log function is the inverse of the exponential function $y = e^x$, it is written as $y = \ln x$, which means $y = \log_e x$				
Laws of Logs	• $\mathbf{\log_a xy = \log_a x + \log_a y}$ **e.g.** $\log_4 8 + \log_4 2 = \log_4(8 \times 2) = \log_4 16 = 2$ (since $4^2 = 16$) • $\mathbf{\log_a \frac{x}{y} = \log_a x - \log_a y}$ **e.g.** $\log_4 8 - \log_4 2 = \log_4 \frac{8}{2} = \log_4 4 = 1$ (since $4^1 = 4$) • $\mathbf{\log_a x^n = n \log_a x}$ **e.g.** $\frac{1}{3}\log_9 27 = \log_9 27^{\frac{1}{3}} = \log_9 3 = \frac{1}{2}$ (since $9^{\frac{1}{2}} = 3$) • $\mathbf{\log_a a = 1}$ **e.g.** $\log_5 5 = 1$ (since $5^1 = 5$)				
Use the Laws of Logs to Solve Log Equations	**Ex 1.** Solve: $\log_5(x+1) + \log_5(x-3) = 1$ **Soln.** $\log_5(x+1)(x-3) = 1$ (using first law) $(x+1)(x-3) = 5$ (since $5^1 = 5$) $x^2 - 2x - 3 = 5$ (solve for *x*) $x^2 - 2x - 8 = 0$ $(x+2)(x-4) = 0 \quad \therefore x = -2,\ x = 4$				

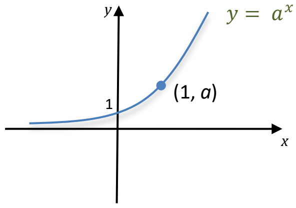
Topic	Skills	Notes			
	Ex 2. Find *x* if $4\log_x 6 - 2\log_x 4 = 1$ **Soln.** $\log_x 6^4 - \log_x 4^2 = 1$ $\log_x \frac{6^4}{4^2} = 1$ $\frac{6^4}{4^2} = x$ $x = \frac{2^4 \times 3^4}{2^4}$ (since $6^4 = 2^4 \times 3^4$ and $4^2 = 16 = 2^4$) $x = 3^4$ $x = 81$				
Use Laws of Logs to Solve Exponential Growth or Decay Problems	• For finding an initial value; substitue given values in to equation to determine the initial value • For finding a half-life, make the equation equal to one half **e.g.** In the equation, where *A* represents micrograms of a radioactive substance remaining over time *t*. Find: (a) the initial value if there are 500 microgram after 100 years (b) the half-life of the substance (a) $A_t = A_0 e^{-0.004t}$ $500 = A_0 e^{-0.004\times 100}$ $500 = 0.67\, A_0$ $A_0 = 746$ micrgrams (b) $373 = 746e^{-0.004t}$ $\frac{1}{2} = e^{-0.004t}$ $\ln\frac{1}{2} = \ln e^{-0.004t}$ $-0.004t = \ln\frac{1}{2}$ $t = 173$ years				
Formulae for Experimental Data	In experimental data questions, two types of exponential functions are considered, $y = kx^n$ and $y = ab^x$ $\boldsymbol{y = kx^n}$ Taking logs of boths sides, this equation may be expressed as $\log y = n \log x + \log k$. To find the unknown values *n* and *k*: • If the data given is *x* and *y* data, then take logs of two sets of the data for *x* and *y* and form a new table with $\log x$ and $\log y$ • Substitute new values into $\log y = n \log x + \log k$ and solve simultaneously to find values for *n* and log *k* • Find *k* by solving log *k* • Write $y = kx^n$ with values of *k* and *n* $\boldsymbol{y = ab^x}$ Taking logs of boths sides, this equation may be expressed as $\log y = x \log b + \log a$. To find the unknown values *a* and *b*: • If the data given is *x* and *y* data, then take logs of the data for *y* • Substitute values into $\log y = x \log b + \log a$ and solve simultaneously to find values for log *a* and log *b* • Find *a* and *b* by solving log *a* and log *b* • Write $y = ab^x$ with values of *a* and *b*				
Sketch the Graph of the Inverse Function of a Log or Exponential Function	See *Graphs of Functions*				

Topic	Skills	Notes			
Addition Formulae					
Prior skills					
Pythagoras Theorem	$a^2 = b^2 + c^2$ 				
SOHCAHTOA	$\sin\theta = \frac{Opp}{Hyp}$, $\cos\theta = \frac{Adj}{Hyp}$, $\tan\theta = \frac{Opp}{Adj}$ 				
Solve Trig Equations	Use the CAST diagram or graphical method to solve equations (see *Relationships* in National 5 checklist)				
Exact Values	See *Trigonometry*				
Convert from Degrees to Radians and Vice-Versa	See *Trigonometry*				
New Skills					
Use Exact Values to Calculate Related Obtuse Angles	**Ex 1.** Find the exact value of cos 225^o **Soln.** The related acute angle is 45^o since 180^o + 45^o = 225^o From the graph or CAST diagram cos 225^o is negative. $\therefore\ \cos 225 = -\cos 45 = -\frac{1}{\sqrt{2}}$ **Ex 2.** Find the exact value of $\sin\frac{2\pi}{3}$ **Soln.** The related acute angle is $\frac{\pi}{3}$ since $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$ From the graph or CAST diagram $\sin\frac{2\pi}{3}$ is positive. $\therefore\ \sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$				
Use Addition Formulae to Expand Expressions	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ **NB:** For sin functions the signs are the same, for cos functions the signs are different				
Use Addition Formulae to Evaluate Exact Values of Expressions	**Ex 1.** Find the exact value of cos 75^o **Soln.** $\cos 75° = \cos(45 + 30)°$ (use addition formulae expansion) $= \cos 45 \cos 30 - \sin 45 \sin 30$ (use exact values) $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}}$ **Ex 2.** Given $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, show that $\sin(A + B) = \frac{56}{65}$ **Soln.** Use SOHCAHTOA to sketch triangles from the info given and use Pythagoras to find unknowns  Expand using Addition Formulae $\sin(A + B) = \sin A\cos B + \cos A\sin B$ $= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}$ $= \frac{56}{65}$				

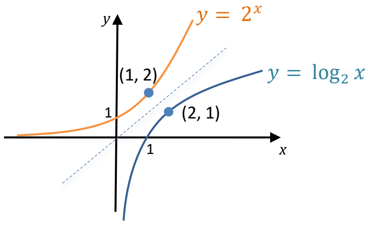
Topic	Skills	Notes			
Solve Trig Equations using Trig Identities	• Determine which part of the equation has a related identity • Replace trigonometric term with related identity and solve **e.g.** Solve $\sin 2x + \sin x = 0$, for $0 \le x \le 180°$ **Soln.** As $\sin 2x = 2\sin x \cos x$, then $2\sin x \cos x + \sin x = 0$, solve by factorising Factorise $\sin x\,(2\cos x + 1) = 0$ $\sin x = 0$ $2\cos x + 1 = 0$ (using graph) $\cos x = -\frac{1}{2}$ (using CAST) $x = 0°, 180°, \cancel{360°}$ $x_A = 60°$ $x = 120°, \cancel{240°}$ $\therefore x = 0°, 120°, 180°$				
Wave Function					
Prior Skills					
Solve Trig Equations	Use the CAST diagram or graphical method to solve equations (see *Relationships* in National 5 checklist)				
Exact Values	See *Trigonometry*				
Use Addition Formulae to Expand Expressions	See *Addition Formulae*				
New Skills					
Write an Expression Form $k\sin(x \pm \alpha)$ or $k\cos(x \pm \alpha)$	$a\cos x + b\sin x$ can be written in one of the following forms: $k\sin(x + \alpha)$ $k\sin(x - \alpha)$ $k\cos(x + \alpha)$ $k\cos(x - \alpha)$ Where $k = \sqrt{a^2 + b^2}$ and $\tan\alpha$ is derived from a and b **Ex 1.** Express $\sqrt{3}\sin x + \cos x$ in the form $k\sin(x + \alpha)°$ where $k > 0$ and $0 \le x \le 360°$ **Soln.** $k\sin(x + \alpha) = k(\sin x \cos\alpha + \cos x \sin\alpha)$ (Expand) $= k\cos\alpha \sin x + k\sin\alpha \cos x$ $= \sqrt{3}\sin x + \cos x$ $\therefore\ k\cos\alpha = \sqrt{3}$ and $k\sin\alpha = 1$ To find k: $k = \sqrt{3+1} = 2$ To find α: $\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$ $\tan\alpha = \frac{1}{\sqrt{3}}$ (use exact values) $\alpha = 30°$ (**NB:** $k\sin\alpha$ and $k\cos\alpha$ are both positive, therefore the angle is in quadrant 1, i.e. less than $90°$) $\therefore\ \sqrt{3}\sin x + \cos x = 2\sin(x + 30)°$				

Topic	Skills	Notes			
	Ex 2. Express $8\cos x - 6\sin x$ in the form $k\cos(x+\alpha)°$ where $k > 0$ and $0 \le x \le 360°$ **Soln.** $k\cos(x+\alpha) = k(\cos x\cos\alpha - \sin x\sin\alpha)$ (Expand) $= k\cos\alpha\cos x - k\sin\alpha\sin x$ $= 8\cos x - 6\sin x$ $\therefore\ k\cos\alpha = 8$ and $k\sin\alpha = 6$ To find k: $k = \sqrt{8^2+6^2} = 10$ To find α: $\tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$ $\tan\alpha = \frac{6}{8}$ (use calculator) $\alpha = 36.9°$ $\therefore\ 8\cos x - 6\sin x = 10\cos(x+36.9)°$				
Graphs of Functions					
Prior Skills					
Sketch Quadratics	Sketch quadratics of the form $y = kx^2$ and $y = a(x+b)^2 + c$				
Sketch Trig Graphs	Sketch graphs of the form $y = a\sin bx + c$ and $y = a\cos bx + c$				
New Skills					
Sketch Related Graphs	Ensure all given coordinates are translated and marked on the new graph and axes and graphs are labelled $y = f(x) + a$ Graph moves up or down by a *Up* for $f(x) + a$ *Down* for $f(x) - a$  $y = f(x+a)$ Graph moves left or right *Left* when $f(x+a)$ *Right* for $f(x-a)$  $y = -f(x)$ Graph reflects in x-axis  $y = f(-x)$ Graph reflects in y-axis  $y = kf(x)$ Graph is stretched vertically for $k > 1$ Graph is squashed vertically for $0 < k < 1$ 				

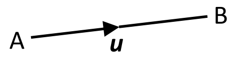
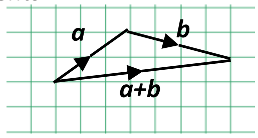
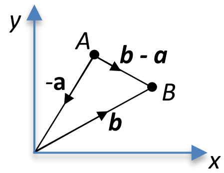
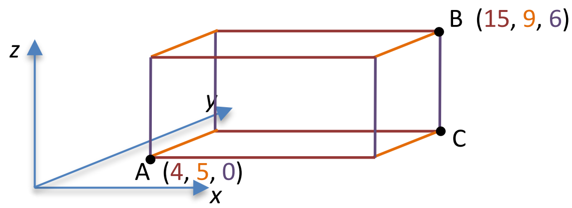
<table>
<tr><th>Topic</th><th>Skills</th><th>Notes</th><th></th><th></th><th></th></tr>
<tr><td></td><td>$y = f(kx)$
Graph is squashed horizontally for $k > 1$
Graph is stretched horizontally for $0 < k < 1$

</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sketch Log and Exponential Graphs</td><td>Log graphs of the form $y = \log_a x$ always cut the x-axis at the point (1, 0) and will pass through (a, 1)


Exponential graphs of the form $y = a^x$ always cut the y-axis at the point (0, 1) and will pass through (1, a)


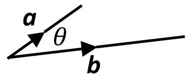
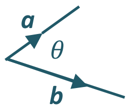
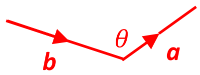
All of the related graph transformations above apply to log and exponential graphs</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sketch the Graph of the Inverse Function of a Log or Exponential Function</td><td>The graph of an inverse function is reflected along the line $y = x$. The logarithmic graph is the inverse of the exponential graph and vice-versa

e.g. For the graph of the function $y = 2^x$ the inverse function is $y = \log_2 x$

</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Sketch a Trig Graph of the Form $y = k\sin(x \pm \alpha)$ or $y = k\cos(x \pm \alpha)$</td><td>See Trigonometry – Sketch a Trig Graph from its Equation</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="6">Sets and Functions</td></tr>
<tr><td colspan="6">Prior Skills</td></tr>
<tr><td>Identify the Turning Point of a Quadratic</td><td>From completed square form $y = a\,(x + b)^2 + c$, turning point is (-b, c)</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="6">New Skills</td></tr>
<tr><td>Find Composite Functions</td><td>Composite functions consist of one function within another.
e.g. If $f(x) = 3x - 2$ and $g(x) = x^2 - 4$,
find (a) $f(g(x))$ (b) $g(f(x))$
Soln.
(a) $f(g(x)) = 3(x^2 - 4) - 2 = 3x^2 - 12 - 2 = 3x^2 - 14$
(b) $g(f(x)) = (3x - 2)^2 - 4 = 9x^2 - 12x + 4 - 4 = 9x^2 - 12x$</td><td></td><td></td><td></td><td></td></tr>
</table>

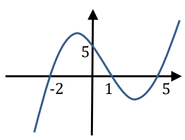
Topic	Skills	Notes			
Evaluate Using Composite Functions	**e.g.** Find $H(-1)$ where $H(x) = g\big(f(x)\big)$ and $f(x) = 3x - 2$, $g(x) = x^2 - 4$ **Soln.** *Method 1:* $H(x) = g\big(f(x)\big) = \ldots = 9x^2 - 12x$ (from example above) $H(-1) = 9(-1)^2 - 12(-1) = 21$ *Method 2:* $f(-1) = 3(-1) - 2 = -5$ $g(-5) = (-5)^2 - 4 = 21$				
Determine a Suitable Domain of a Function	Restrictions on the domain of a function occur in two instances at Higher Mathematics. A restriction will occur when a denominator is zero, which is undefined or when a square root is negative, which is non-real **e.g.** For $f(x) = \frac{12x}{(4-x)^2}$ and $x \in \boldsymbol{R}$, write a restriction on the domain of $f(x)$ **Soln.** $x \neq 4$ as this would make the denominator zero				
State the Range of a Function	**e.g.** State the minimum turning point of the function $f(x) = x^2 + 5$ and hence state the range of the function (see *prior skills*) **Soln.** Minimum turning point is (0, 5) as the y-coordinate of the turning point is 5, the range of the function is $f(x) > 5$				
Find an Inverse Function	For a function $f(x)$ there is an inverse function $f^{-1}(x)$, such that $\boldsymbol{f\big(f^{-1}(x)\big) = x}$ To find an inverse function: • Replace *x* with *y* in the function and $f(x)$ with *x* • Change the subject to *y* **e.g.** For the function $f(x) = \frac{3}{4-x^2}$ find the inverse function $f^{-1}(x)$ **Soln.** $f(x) = \frac{3}{4-x^2}$ $x = \frac{3}{4-y^2}$ $4 - y^2 = \frac{3}{x}$ $4 - \frac{3}{x} = y^2$ $y = \sqrt{4 - \frac{3}{x}}$ $\therefore f^{-1}(x) = \sqrt{4 - \frac{3}{x}}$				
Common Terms					
Domain	The *domain* of a function is the set of numbers that can be input into the function (*see Determine a Suitable Domain* above)				
Range	The *range* of a function is what comes out of the function after the x-values have been put in (see *State the Range of a Function*)				
Number Sets	There are five standard number sets to consider at Higher Mathematics • The set of *natural numbers* $\boldsymbol{N}$ (counting numbers) $\boldsymbol{N} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \ldots\}$ • The set of *whole numbers* $\boldsymbol{W}$ (the same as *natural numbers*, but inclusive of zero) $\boldsymbol{W} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \ldots\}$ • The set of *integers* $\boldsymbol{Z}$ (the same as *whole numbers*, but inclusive of negative numbers) $\boldsymbol{Z} = \{\ldots, \mathbf{-2}, \mathbf{-1}, \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \ldots\}$				

Topic	Skills	Notes			
	• The set of *rational numbers* $\boldsymbol{Q}$ (all numbers that can be written as fractions) • The set of *real numbers* $\boldsymbol{R}$ (inclusive of both rational and irrational numbers i.e. $\pi, \sqrt{3}$, etc.)				
Vectors					
Prior Skills					
Vector Notation	Vectors can be named in one of two ways. Either by using the letters of the points at the end of the line segment $\overrightarrow{AB}$ or by using a single letter in lower case u. When writing the lower case name, underline the letter 				
2D Line Segments	Add or subtract 2D line Segments • Vectors end-to-end • Arrows in same direction 				
Finding a Vector from Two Coordinates	Know that to find a vector between two points A and B then $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ **NB:** Vector notation for a vector between two points A and B is $\overrightarrow{AB}$ 				
3D Vectors	Determine coordinates of a point from a diagram representing a 3D object Look at difference in x, y and z axes individually **e.g.** The cuboid is parallel to the *x*, *y* and *z* axes. Find the coordinates of C  C (15, 9, 0)				
Position Vectors	The position vector of a coordinate is the vector from the origin to the coordinate. **E.g.** A (4, -3) has the position vector $\boldsymbol{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$				
The Zero Vector	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is called the zero vector, written **0** $\boldsymbol{u} + (-\boldsymbol{u}) = \boldsymbol{0}$				
Add and Subtracy Vector Components	Add and Subtract 2D and 3D vector components. $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ $\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} 1+3 \\ 1+2 \\ 4+5 \end{pmatrix}$				
Multiply Vector Components	Multiply vector components by a scalar $2\boldsymbol{a} = 2\begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 8 \end{pmatrix}$				

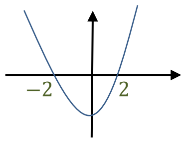
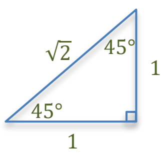
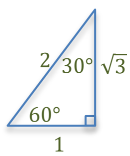
Topic	Skills	Notes			
Find the Magnitude of a vector	Find the magnitude of a 2D or 3D vector: For vector $\boldsymbol{u} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\lvert\boldsymbol{u}\rvert = \sqrt{\boldsymbol{x^2 + y^2}}$ For vector $\boldsymbol{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\lvert\boldsymbol{v}\rvert = \sqrt{\boldsymbol{x^2 + y^2 + z^2}}$				
New Skills					
Writing Vectors	Vectors can be written in component form i.e. $\boldsymbol{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or in terms of $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$,where each of these represents the unit vector in the *x*, *y* and *z* direction. **e.g.** $\overrightarrow{\mathrm{AB}} = 4\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k}$can be written as $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$				
Parallel Vectors	Vectors are parallel if one vector is a scalar multiple of the other **e.g.** $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4 \\ 4 \\ 16 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ $\boldsymbol{b} = 4\boldsymbol{a}$ ∴ vectors are parallel				
The Unit Vector	For any vector, there is a parallel vector $\boldsymbol{u}$ of magnitude 1. This is called the unit vector **e.g.** Find the unit vector $\boldsymbol{u}$ parallel to vector $\boldsymbol{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $\lvert\boldsymbol{a}\rvert = \sqrt{5^2 + 12^2} = 13$ $\therefore \quad \boldsymbol{u} = \frac{1}{13}\begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 5/13 \\ 12/13 \end{pmatrix}$				
Collinearity	Points are said to be collinear if they line on the same line. To show points are collinear using vectors; show (a) they are parallel by demonstrating one vector is a scalar multiple of the other and (b) that they share a common point **e.g.** Show that A(-3, 4, 7), B(-1, 8, 3) and C(0, 10, 1) are collinear **Soln.** $\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{\mathrm{BC}} = \boldsymbol{c} - \boldsymbol{b} = \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\overrightarrow{\mathrm{AB}} = 2\overrightarrow{\mathrm{BC}}$ and point B is common ∴ A, B and C are collinear				
Divide Vectors into a Given Ratio to Find an Unknown Point	**e.g.** P is the point (6, 3, 9) and R is (12, 6, 0). Find the coordinates of Q, such that Q divides PR in the ratio 2:1 **Soln.** $\frac{\overrightarrow{\mathrm{PQ}}}{\overrightarrow{\mathrm{QR}}} = \frac{2}{1}$ $\overrightarrow{\mathrm{PQ}} = 2\overrightarrow{\mathrm{QR}}$ $\boldsymbol{q} - \boldsymbol{p} = 2(\boldsymbol{r} - \boldsymbol{q})$ $\boldsymbol{q} - \boldsymbol{p} = 2\boldsymbol{r} - 2\boldsymbol{q}$ $3\boldsymbol{q} = 2\boldsymbol{r} + \boldsymbol{p}$ $3\boldsymbol{q} = 2\begin{pmatrix} 12 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 30 \\ 15 \\ 9 \end{pmatrix}$ $\boldsymbol{q} = \begin{pmatrix} 10 \\ 5 \\ 3 \end{pmatrix}$ ∴ Q (10, 5, 3) **NB:** this could also be calculated using section formula				

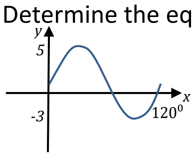
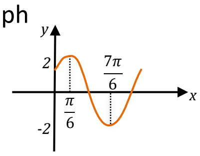
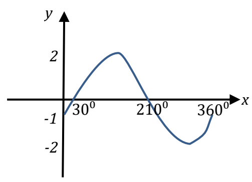
Topic	Skills	Notes			
Find the Ratio in which a Point Divides a Line Segment	**e.g.** A(-2, -1, 4), B(1, 5, 7) and C(7, 17, 13) are collinear. What is the ratio in which B divides AC? **Soln.** $\overrightarrow{\mathrm{AB}} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix}1\\5\\7\end{pmatrix} - \begin{pmatrix}-2\\-1\\4\end{pmatrix} = \begin{pmatrix}3\\6\\3\end{pmatrix}$ $\overrightarrow{\mathrm{BC}} = \boldsymbol{c} - \boldsymbol{b} = \begin{pmatrix}7\\17\\13\end{pmatrix} - \begin{pmatrix}1\\5\\7\end{pmatrix} = \begin{pmatrix}6\\12\\6\end{pmatrix} = 2\begin{pmatrix}3\\6\\3\end{pmatrix}$=$2\overrightarrow{AB}$ $2\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{BC}}$ $\frac{\overrightarrow{\mathrm{AB}}}{\overrightarrow{\mathrm{BC}}} = \frac{1}{2}$ $\therefore \overrightarrow{\mathrm{AB}}:\overrightarrow{\mathrm{BC}}$ 1:2				
Scalar Product	When given an angle between two vectors, the scalar product is calculated using $\boldsymbol{a}.\boldsymbol{b} = \ \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \cos\theta$  **NB:** To find the angle between the two vectors θ, the vectors must be pointing away from each other and $0 \ \leq \theta \leq 180°$ When given component form, i.e. if $\boldsymbol{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, the scalar product is calculated using $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$				
Angle Between Two Vectors	The angle between two vectors is calculated by rearranging the scalar product formula $\cos\theta = \frac{\boldsymbol{a}.\boldsymbol{b}}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert}$ which can be expanded to $\cos\theta = \frac{a_1b_1+a_2b_2+a_3b_3}{\lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert}$ **NB:** to find the angle between two vectors, the vectors must be pointing away from or towards each other. They must not be going in the same direction **e.g.**  **Not** 				
Perpendicular Vectors	Vectors are perpendicular when $\boldsymbol{a}.\boldsymbol{b} = 0$				
Common Terms					
Vector	A vector is a quantity that contains both a size and a direction				

Relationships & Calculus

Topic	Skills	Notes			
Polynomials					
Prior Skills					
Factorise	Common Factor, Difference of Two Square and Trinomial factorising				
Solving Quadratics	Algebraically or using quadratic formula				
Factors and Roots	• A root is a value for which a polynomial f(x) = 0. These are the *x*-coordinates at the point of intersection with the x-axis • A factor is the form from which a root is derived **e.g.** for $x(x - 4) = 0$, x and (x – 4) are ***factors***, x = 0 and x = 4 are ***roots***				
New Skills					
Fully Factorise a Polynomial	Use synthetic division **e.g.** Factorise $f(x) = x^3 - 3x + 2$ **Soln.** Set up synthetic division using coefficients from polynomial. - If there is no term, use 0 - The value outside the division is derived from factors of the last term (in this case factors of 2) - If the remainder of the division is 0 then the value outside the division is a root -2 | 1 0 -3 2 -2 4 -2 1 -2 1 | 0 ∴ $(x + 2)$ is a factor and x = -2 is a root $(x + 2)(x^2 - 2x + 1) = 0$ $(x + 2)(x - 1)(x - 1) = 0$, ∴ x = -1 (twice) and x = 2				
Remainder Theorem	Find the Quotient and Remainder of a Function • Use synthetic division with the given value • The value at the end is the remainder **e.g.** Find the quotient and remainder when $f(x) = (x^3 - 3x + 9)$ is divided by $(x - 3)$ **Soln.** 3 | 1 0 -3 9 3 9 18 1 3 6 | 27 ∴ $f(x) = (x^3 - 3x + 9) = (x - 3)(x^2 + 3x + 6) + 27$ Quotient is $(x^2 + 3x + 6)$ and remainder 27				
Identify the Equation of a Polynomial from a Graph	• Determine the factors from the roots on the graph • Set up a polynomial with a coefficient of k outside the brackets • Substitute the y-intercept or given point to determine value of k **e.g.**  ***Soln.*** $y = k(x + 2)(x - 1)(x - 5)$, When x = 0, y = 5 $5 = k(0 + 2)(0 - 1)(0 - 5)$ $5 = 10k$, $k = \frac{1}{2}$ ∴ $y = \frac{1}{2}(x + 2)(x - 1)(x - 5)$				

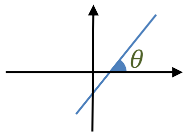
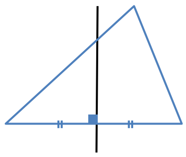
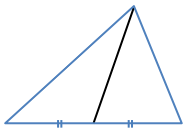
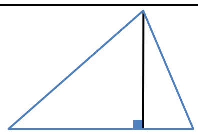
Topic	Skills	Notes			
Show that a Term is a Factor of a Polynomial	• From the factor, determine the root (*see Factors and Roots above*) • Use synthetic division with the given value If the remainder is 0 the term is a factor				
Find Unknown Coefficients of a Polynomial	• Substitute roots into equation and solve simultaneously **e.g.** Find the values of *p* and *q* if $(x+2)$ and $(x-1)$ are factors of $f(x) = x^3 + 4x^2 + px + q$ **Soln.** $(x+2)$ is a factor $\therefore x = -2$ is a root $(-2)^3 + 4(-2)^2 - 2p + q = 0$ $8 - 2p + q = 0$ $q = 2p - 8$ $(x-1)$ is a factor $\therefore x = 1$ is a root $(1)^3 + 4(1)^2 + p + q = 0$ $5 + p + q = 0$ $q = -p - 5$ Solve two equations simultaneously $2p - 8 = -p - 5$ … … $\therefore p = 1, q = -6$ and $f(x) = x^3 + 4x^2 + x - 6$				
Sketch the Graph of a Polynomial Function	• Find the *x*-intercepts (roots, when $y = 0$) using synthetic division • Find the *y*-intercept (when $x = 0$) • Find stationary points and their nature (*see differentiation*) • Find large negative and positive *x*				
Common Terms					
Polynomial	A function containing multiple terms of different powers **e.g.** $x^4 - 2x^3 + 3$				
Quadratic Functions					
Prior Skills (see *Relationships* in National 5 Checklist)					
Solve a Quadratic	Graphically, Algebraically or using Quadratic Formula				
Sketch a Quadratic	• In completed square form • In root form				
Identify the Equation of a Quadratic from its Graph	See *Identify the equation of a Polynomial from a Graph* (above)				
Complete the Square	$x^2 + ax + b = (x + \frac{a}{2})^2 + b - a^2$ **Ex 1.** $x^2 + 8x - 13 = (x + 4)^2 - 13 - 16 = (x + 4)^2 - 29$ **Ex 2.** $x^2 + 3x + 10 = (x + \frac{3}{2})^2 + 10 - \frac{9}{4} = (x + \frac{3}{2})^2 + \frac{40}{4} - \frac{9}{4} = (x + \frac{3}{2})^2 + \frac{31}{4}$				
Discriminant	$b^2 - 4ac$ where $y = ax^2 + bx + c$ The discriminant describes the nature of the roots $b^2 - 4ac > 0$ two real roots $b^2 - 4ac = 0$ equal roots (i.e. tangent) $b^2 - 4ac < 0$ no real roots				

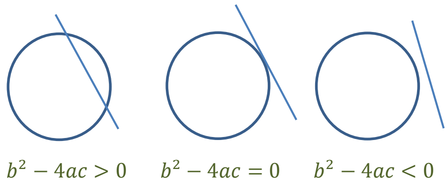
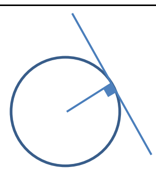
Topic	Skills	Notes			
New Skills					
Show a Line is a Tangent to a Quadratic Function	• Equate the line and quadratic • Bring to one side • Use the discriminant *or* factorise to show repeated root **e.g.** Show that the line $y = x + 5$ is a tangent to the curve $y = x^2 + 5x + 9$ **Soln.** $x^2 + 5x + 9 = x + 5$ $x^2 + 4x + 4 = 0$ $(x + 2)(x + 2) = 0$ $x = -2$ twice ∴ line is a tangent to quadratic.				
Determine Nature of Intersection Between a Line and a Quadratic Function	• Equate line and quadratic • Bring to one side • Use the discriminant to determine nature of intersection				
Determine Points of Intersection between a Line and a Quadratic Function	• Equate line and quadratic • Bring to one side • Solve for *x* • Substitute *x* values into line to find *y* values				
Use Discriminant to Find Unknown Coefficients of a Quadratic Function	• Identify coefficients *a, b* and *c* • Use discriminant **Ex 1.** Find *p* given that $x^2 + x + p = 0$ has real roots **Soln.** *a* = 1, *b* = 1, *c* = p $b^2 - 4ac \geq 0$ $1^2 - 4(1)(p) \geq 0$ $1 - 4p \geq 0$ $1 \geq 4p$ $p \leq \frac{1}{4}$ **Ex 2.** Find *p* given that $4x^2 + 2px + 1 = 0$ has no real roots **Soln.** *a* = 4, *b* = 2*p*, *c* = 1 $b^2 - 4ac < 0$ $(2p)^2 - 4(4)(1) < 0$ $4p^2 - 16 < 0$ $4(p^2 - 4) < 0$ $4(p + 2)(p - 2) < 0$ (Sketch a graph)  For no real roots $-2 < p < 2$				
Common Terms					
Parabola	The graph of a quadratic function				
Trigonometry					
Prior Skills					
Exact Values	Know exact values from table or using triangles and SOHCAHTOA   Know exact values of 0^o, 90^o, 180^o, 270^o and 360^o from trig graphs				
Solve Trig Equations	Use the CAST diagram or graphical method to solve equations (see *Relationships* in National 5 checklist)				

Topic	Skills	Notes			
Determine Period, Shape and Max and Min Values from a Trig Equation	$y = a\cos bx$ Amplitude = *a* Period = $\frac{360}{b}$ **NB:** in the graph $y = a\tan bx$ the amplitude cannot be measured				
Adding Fractions	For solutions in radians, the ability to add fractions is required **e.g.** $\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$, $2\pi + \frac{\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3} = \frac{7\pi}{3}$				
New Skills					
Convert from Degrees to Radians	• Multiply degrees by π and divide by 180 then simplify **e.g.** Change 35° to radians **Soln.** $\frac{35\pi}{180} = \frac{7\pi}{36}$				
Convert from Radians to Degrees	• Multiply radians by 180 and divide by π then simplify **e.g.** Change $\frac{5\pi}{6}$ to degrees **Soln.** $\frac{5\pi}{6} = \frac{5\times180\pi}{6\pi} = \frac{5\times30}{1} = 150°$				
Solve Trig Equations with multiple solutions	• Identify how many solutions from the question • Solve the equation **e.g.** Solve $2\cos 3x = 1$, for $0 \le x \le \pi$ **Soln.** $2\cos 3x = 1$ $\cos 3x = \frac{1}{2}$ (As $\cos 3x$ has 6 solutions for $0 \le x \le 2\pi$, ∴ 3 solutions for $0 \le x \le \pi$) $3x = \frac{\pi}{3}, \pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$ $3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}$ $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}$				
Solve Trig Equations by Factorising	Factorise the equation in the same way as an algebraic equation, by looking for *common factors, difference of two square* or a *trinomial* **e.g.** Solve $2\sin x\cos x + \sin x = 0$, for $0 \le x \le 180°$ **Soln.** Factorise $\sin x\,(2\cos x + 1) = 0$ $\sin x = 0$ $2\cos x + 1 = 0$ $x = 0°, 180°, ~~360°~~$ $\cos x = -\frac{1}{2}$ (using CAST) $x_A = 60°$ $x = 120°, ~~240°~~$ $\therefore x = 0°, 120°, 180°$				
Identify the Equation of a Trig function from its Graph	**e.g.** Determine the equation of the graph **1.**  **Ans:** $y = 4\sin 3x + 1$ **2.**  **Ans:** $y = 2\cos(x - \frac{\pi}{6})$				
Sketch a Trig Graph from its Equation	**e.g.** Sketch the graph of $y = 2\sin(x - 30)°$ for $0 \le x \le 360$ showing clearly where the graph cuts the *x*-axis and the *y*-axis **Ans.** Amplitude of 2. Graph moves 30° to the right. Find x-intercepts when *y* = 0. Find *y*-intercept when *x* = 0 				

Topic	Skills	Notes			
Common Terms					
Radians	Radians are an alternative unit for measuring angles. π radians = 180^o Common Conversions: $\frac{\pi}{2} = 90°$, $\frac{\pi}{3} = 60°$, $\frac{\pi}{4} = 45°$, $\frac{\pi}{6} = 30°$, $\frac{2\pi}{3} = 120°$, $\frac{3\pi}{2} = 270°$				
Further Calculus					
Prior Skills					
Differentiate	See *Differentiation* (in *Applications*)				
Integrate	See *Integration* (in *Applications*)				
Trig Equations	See *Trigonometry* (above)				
Radians	See *Trigonometry* (above)				
New Skills					
Differentiate Trig Functions	$y = \sin x$ $\frac{dy}{dx} = \cos x$ $y = \cos x$ $\frac{dy}{dx} = -\sin x$				
Integrate Trig Functions	$\int \sin x \, dx$ $= -\cos x + C$ $\int \cos x \, dx$ $= \sin x + C$				
Chain Rule	Used for differentiating composite functions • Differentiate the outer function • Multiply by the derivative of the inner function e.g. If $h(x) = f(g(x))$ then $h'(x) = f'(g(x)) \times g'(x)$ **Ex. 1.** Find $\frac{dy}{dx}$ when $y = \sqrt{2x-5}$ **Soln.** Prepare function for differentiation $y = (2x-5)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(2x-5)^{-\frac{1}{2}} \times 2$ $\frac{dy}{dx} = \frac{1}{\sqrt{2x-5}}$ **Ex 2.** Find $\frac{dy}{dx}$ when $y = 3\cos^2 x$ **Soln.** Prepare function for differentiation $y = 3(\cos x)^2$ $\frac{dy}{dx} = 6(\cos x)^1 \times \sin x$ $\frac{dy}{dx} = 6\cos x \sin x$				
Integration of Composite Functions	When integrating composite functions • Integrate the outer function • Divide by the <u>derivative</u> of the inner function e.g. $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{n+1 \times a} + C$ **Ex 1.** $\int (2x^3+5)^4 \, dx$ **Soln.** $\int (2x^3+5)^4 \, dx = \frac{(2x^3+5)^5}{5 \times 6x^2} + C = \frac{(2x^3+5)^5}{30x^2} + C$ **Ex 1.** $\int \sin(4x-3) \, dx$ **Soln.** $\int \sin(4x-3) \, dx = \frac{-\cos(4x-3)}{4} + C$				

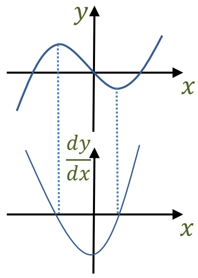
Applications

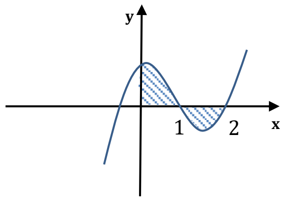
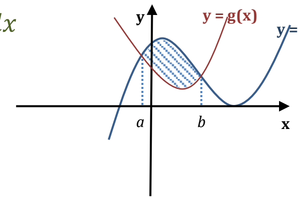
Topic	Skills	Notes			
The Straight Line					
Prior Skills					
Distance Between Two Points	Distance formula or alternative: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$				
Gradient of a line	• $m = \frac{y_2 - y_1}{x_2 - x_1}$ • $m = \tan\theta$ • Perpendicular gradients: $m_1 \times m_2 = -1$  **e.g.** if $m_1 = 2$, the perpendicular gradient $m_2 = -\frac{1}{2}$				
Equation of a Line From Two Points	For every equation of line a **point** and **gradient** is required • Calculate gradient and substitute point (*a, b*) into equation: $y - b = m(x - a)$ • Expand bracket and simplify				
Midpoint	$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$				
Point of Intersection	• Solve using simultaneous equations				
New Skills					
Equation of a Perpendicular Bisector	• Find the midpoint of the line joining the two points • Find gradient using perpendicular gradients • Substitute midpoint and inverted gradient into $y - b = m(x - a)$ 				
Equation of a Median	• Find the midpoint of the line joining the two points • Find gradient of the median • Substitute into $y - b = m(x - a)$ 				
Equation of an Altitude	• Find gradient of the altitude using perpendicular gradients • Substitute into $y - b = m(x - a)$ with point from vertex 				
Collinearity	• Show that three points are collinear (i.e. on the same line) • Find gradients (the same if parallel) and point in common • Statement: Points A, B and C are collinear as $m_{AB} = m_{BC}$ and point B is common to both				
Common Terms					
Collinear	Points on the same line				
Congruent	The same size				
Concurrent	Lines that intersect at the same point **NB:** In a triangle, altitudes are concurrent (intersect at *orthocentre*), medians are concurrent (intersect at *centroid*) and perpendicular bisectors are concurrent (intersect at *circumcentre*)				
Centroid	The point of intersection of the three medians of a triangle				
Circumcentre	The point of intersection of three perpendicular bisectors in a triangle				
Orthocentre	The point of intersection of three altitudes of a triangle				

Topic	Skills	Notes			
The Circle					
Prior Skills					
Distance between two points	Distance formula or alternative: $$Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$				
Gradient	• $m = \frac{y_2 - y_1}{x_2 - x_1}$ • Perpendicular gradients: $m_1 \times m_2 = -1$				
Midpoint	$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$				
Discriminant	• $b^2 - 4ac$ where $ax^2 + bx + c = 0$ 				
New Skills					
Equation of Circle with Centre the Origin and Radius *r*	• $x^2 + y^2 = r^2$				
Equation of a Circle with Centre (*a, b*) and Radius *r*	• Determine the centre and radius **NB:** Finding the centre often involves finding a midpoint of a diameter, or using a coordinate diagram and symmetry • Substitute into equation $(x - a)^2 + (y - b)^2 = r^2$				
Centre and Radius of a Circle from its Equation	• Use the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ • Centre from $(-g. -f)$ • Radius: $r = \sqrt{g^2 + f^2 - c}$ **NB:** if $g^2 + f^2 - c < 0$ the equation is not a circle				
Equation of a Tangent to a Circle	• Determine the gradient of the radius from centre and the point of contact of the tangent • Find gradient using perpendicular gradients • Substitute perpendicular gradient and point into equation of a line $y - b = m(x - a)$ 				
Points of Intersection of a Line and a Circle	• Rearrange the line to $y = mx + c$ then substitute into the equation of a circle • Solve the quadratic to find *x* • Substitute *x* value into $y = mx + c$ to find *y*				
Use Discriminant to Determine whether a Line and Circle Intersect	• Rearrange the line to $y = mx + c$ then substitute into the equation of a circle • Simplify to quadratic form • Use discriminant to determine intersection				
Common Terms					
Concentric	Circles that have the same centre				
Recurrence Relations					
Prior Skills					
Finding Percentage Multipliers	• Determine whether question is percentage increase or decrease • Add or subtract from 100% • Divide by 100 **e.g.** 4.3% increase 100% + 4.3% = 104.3% = 1.043				

Topic	Skills	Notes			
New Skills					
Form Linear Recurrence Relations	• Find values of *a* and *b* for relation $u_{n+1} = au_n + b$ where *a* is the percentage multiplier and *b* is the increase				
Use Linear Recurrence Relations to Find Values	• Start with u_0 (initial value) and substitute into relation **NB:** Set up calculator by inputting u_0 and pressing '=' then $ANS \times a + b$ and continue pressing '=' until answer is reached. Write down all answers				
Find the Limit of a Linear Recurrence Relation	• Determine values of *a* and *b* • Ensure -1 < *a* < 1 • Use limit formula $L = \frac{b}{1-a}$ • Interpret what the limit means in a specific context				
Differentiation					
Prior Skills					
Laws of indices	• Know and use each of the laws of indices to manipulate algebraic fractions; **e.g.** $\frac{x^3+\sqrt{x}}{x^2} = \frac{x^3}{x^2} + \frac{x^{\frac{1}{2}}}{x^2} = x + x^{-\frac{3}{2}}$				
New Skills					
Differentiate a Function	• To differentiate $f(x) = ax^n$, $f'(x) = anx^{n-1}$ • For $f(x) = g(x) + h(x)$ $f'(x) = g'(x) + h'(x)$ **NB:** For differentiation questions, algebraic fractions need to be broken down into individual fractions (*see Laws of Indices above*)				
Find the Gradient or Rate of Change of a Function at a Given Point	• Know that ***f'*(x) = *m* = rate of change** • Differentiate the function • Substitute x-coordinate into derivative ***e.g.*** *Find the gradient of* $f(x) = 2x^3$ *when x = 1* ***Soln.*** $f'(x) = 6x^2$ $f'(4) = 6(1)^2 = 6$				
Find the Equation of a Tangent to a Curve	• Differentiate function • Find gradient from derivative (*see above*) • Substitute point and gradient into equation of a line $y - b = m(x - a)$				
Find Stationary Points and Determine their Nature	*Find Stationary Points* • Differentiate function • Know that stationary points occur when $f'(x) = 0$ • Solve $f'(x) = 0$ to find *x*-coordinates of stationary points • Substitute x-coordinates into *f*(x) to find y-coordinates *Determine Nature* • Draw nature table with x values slightly above and below the stationary points (make sure they are not lower or higher than any other stationary points) • Sketch the nature from positive or negative values • Answer question, **e.g.** Max turning point at (x, y)				

x	3-	3	3+
f'(x)	-	0	+
slope	\	_	/

Topic	Skills	Notes			
Sketch a curve	• Find stationary points and nature (*see above*) • Find roots by solving function (when y = 0) • Find y-intercept (when x=0) • Find large positive and large negative x. **e.g.** $as\ \boldsymbol{f(x)} \to -\infty, \boldsymbol{x} \to -\infty$ and as $\boldsymbol{f(x)} \to +\infty, \boldsymbol{x} \to +\infty$ • Sketch information on graph				
Sketch the derived function	• Sketch function • Extend stationary points to other coordinate axis • Determine where the gradient *m* is +ve and –ve. (the gradient is +ve where the graph of $\frac{dy}{dx}$ is above the x-axis and negative where it is below 				
Closed Intervals	Find the maximum and minimum value in a closed interval • Find the stationary points and determine their nature (*see above*) • Find the *y*-coordinates at the extents of the interval • Examine to see where the maximum and minimum values are				
Increasing and Decreasing Functions	• Differentiate the function • Determine where gradient is positive or negative from the derivate or a sketch of the derivative **NB:** A function is increasing where the gradient *m* is positive and decreaseing where the gradient is negative				
Common Terms					
Leibniz Notation	y and $\frac{dy}{dx}$				
Function Notation	$f(x)$ and $f'(x)$				
Rate of Change	The rate at which one variable changes in relation to another. To find rate of change, differentiate function				
Integration					
Prior Skills					
Laws of indices	See *Differentiation*				
New Skills					
Integrate a Function	To integrate: $\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C$ • **NB:** When the integral is indefinite (i.e. there are no limits), remember 'C' the constant of integration				
Evaluate a Definite Integral	• Integrate function • Evaluate between two limits **e.g.** Evaluate $\int_1^3 4x\,dx$ **Soln.** $\int_1^3 4x\,dx = [2x^2]_1^3 = (2(3)^2) - (2(1)^2) = 16$				

Topic	Skills	Notes			
Area Between Curve and x-axis	• Find the area above the x-axis • Find the area below the *x*-axis (ignore the negative) • Add them together $\int_0^1 f(x)\,dx$ and $\int_1^2 f(x)\,dx$ **NB:** The area below the x-axis will give a negative answer. Ignore the negative 				
Area Between Two Curves	• Set the curves equal to each other and solve to find the limits • Set up integral with: $\int [upper\ curve - lower\ curve]\ dx$ $\int_a^b [f(x) - g(x)]\ dx$ • Evaluate answer 				
Differential Equations	Equations of the form $\frac{dy}{dx} = ax + b$ are called *differential equations.* They are solved by integration **e.g.** The curve $y = f(x)$ is such that $\frac{dy}{dx} = 9x^2$, the curve passes through (1, 5). Express *y* in terms of *x* **Soln.** $y = \int 9x^2\,dx = 3x^3 + C$ at (1, 5), $5 = 3(1)^3 + C$ $5 = 3 + C$ $C = 2 \quad \therefore \quad y = 3x^3 + 2$				

